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RECENT DEVELOPMENTS IN MULTILEVEL OPTIMIZATION

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Figure 1 identifies the general nature of the multidiscipline design task. The key point is that there are relatively few system level design variables, while there may be a great many subsystem design variables. For example, the overall length and diameter of the fuselage, the thickness, aspect ratio, sweep, etc. of the wing, and the maximum thrust may generally define the design of an aircraft as a system. On the other hand, the design of a subsystem such as a wing, consists of hundreds or even thousands of variables defining the aerodynamic shape, skin thickness distribution, spars, webs, etc. Also, this subsystem may be considered to be itself a collection of subsystems, including aerodynamics, structures, controls, hydraulics and others. There is seldom a clear mathematical structure to the overall design task which would make it amenable to efficient solution techniques such as are available for many structural subsystem design problems. Also, the analysis tools for the various components range from purely experimental to empirical to formal solution of the governing equations by finite element or finite difference methods. In view of these complexities, it must be said at the outset that formal multidiscipline optimization is a technology that is still in its infancy.

FEATURES OF THE MULTIDISCIPLINE PROBLEM

RELATIVELY FEW SYSTEM DESIGN VARIABLES

OFTEN COMPLEX/EXPENSIVE ANALYSIS

ANALYTIC GRADIENTS ARE SELDOM AVAILABLE

THERE IS NO CLEAR MATHEMATICAL STRUCTURE

OPTIMIZATION IS TYPICALLY SEEN AS A "BLACK BOX"

FIGURE 1

A key element in engineering design is the use of approximations to develop and solve the analysis/design task. These approximations may be very simple, such as empirical estimates of component weights based on historical data or they may be quite sophisticated such as the formal solution of the Navier Stokes equations. The motivation is usually to provide the efficiency necessary to the real design environment. Figure 2 lists some of the motivations for making approximations. It is noteworthy that in the relatively well developed subsystem field of structural optimization, the technology was pursued for over fifteen years before a formal approach to creating high quality approximations was developed. In other areas such as aerodynamic or propulsion system optimization, this has yet to be pursued to a significant extent.

APPROXIMATIONS

AT THE SUBSYSTEM LEVEL

PROVIDE NECESSARY EFFICIENCY

IN STRUCTURAL OPTIMIZATION

LINEARIZATION WITH RESPECT TO SOME INTERMEDIATE VARIABLES

AT THE SYSTEM LEVEL

DEAL WITH SUBSYSTEM RESPONSES

ACCOUNT FOR INTERACTIONS AMONG SUBSYSTEMS

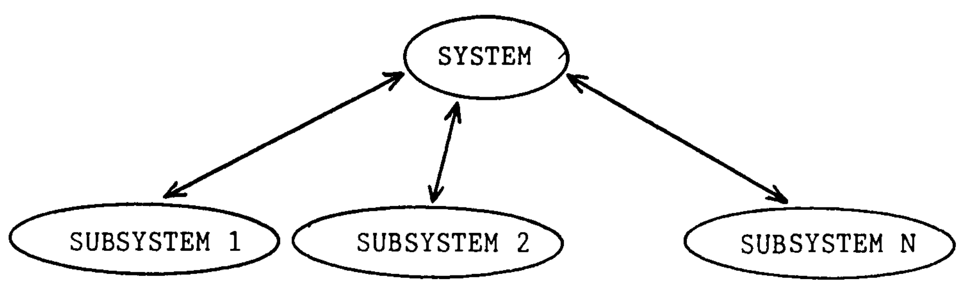
SEND SYSTEM LEVEL INFORMATION TO SUBSYSTEMS

SEND INTERACTION INFORMATION TO SUBSYSTEMS

FIGURE 2

Figure 3 depicts the structure of the formal multidiscipline optimization problem. It is noteworthy that this is a tree structure similar to the general design environment where the system may be thought of as the chief designer and the subsystems as engineering departments. Whenever the system variables are changed, the effects on the subsystems must be accounted for. Similarly, when the subsystem variables are changed, the effect of the overall system must be considered. Subsystems may be defined along discipline lines or by other criteria. For example, the design of a wing may be considered as a subsystem including aerodynamic, structural and other considerations. Alternatively, aerodynamics and structures may be considered to be separate subsystems or lower level subsystems within the general category of wing design. It is clear that aerodynamics and structures play interacting and competing roles in the overall design and so these interactions must be properly accounted for via the system design control. Ideally, aerodynamic and structural design must be done simultaneously. However, this is counter to the usual division along discipline lines and so little emphasis has been directed toward the combined design process, even at the research level.

FORMAL MULTIDISCIPLINE OPTIMIZATION



ADDITIONAL LEVELS OF SUBSYSTEMS MAY EXIST

FIGURE 3

Figure 4 presents a simple cantilevered beam which demonstrates the concepts of multilevel design. The objective is to minimize the material volume subject to limits on the deflection at the beam junction and at the tip, and on the maximum bending stresses and height to width ratios of the members. The design variables of interest are the width, B_i, and height, H_i, of each element, and the length, L_1 ($L_2 = L - L_1$). Clearly, for such a simple problem, this would be solved directly. However, for demonstration purposes, it is possible to formulate it as a multilevel problem with a system level and two subsystems.

The system level problem may be stated as, find the beam length, L_1, and dimensions B_1, H_1, B_2 and H_2 to minimize the volume subject to constraints on the deflections. Additionally, in the present method, subsystem constraints will be imposed, in linearized form, on the stresses and the height to width ratio on the members. Each member can be taken as a subsystem and, during subsystem optimization, the member volume will be minimized subject to constraints on the member stresses and height to width ratio. At this level, the purely system level design variable, L_1, will be held fixed, but the system level constraints (deflections) will be included in linearized form.

Note that, at the system level, all design variables are included. At the subsystem, the design variables that are important to the subsystem are considered, but the strictly system level variable, L_1, is held fixed.

Because of the interdependence between the system and subsystem variables, each level will affect the other. The key issue is how to account for these interactions and how to account for competition between subsystems.

MULTILEVEL DESIGN OF A CANTILEVER BEAM

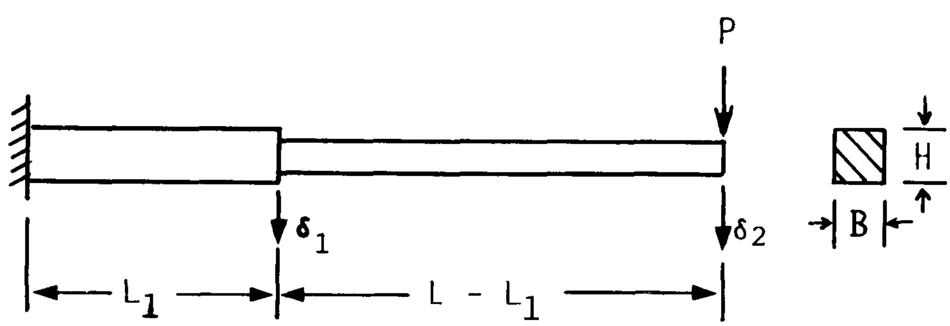


FIGURE 4

A variety of methods have been proposed to deal with the multidiscipline design task in a formal way. Figure 5 presents a recent method developed in an effort to simplify the overall process while maintaining the traditional separation of disciplines [1]. Initially, all system level functions (objective and constraints) are linearized. Each of the subsystem optimizations is then performed, presumably in parallel. At the optimum for the subsystem, its constraints (or a critical and near critical subset) are linearized and returned to the system. The system level optimization is then performed, including these linearized subsystem constraints. Also, at this point, the subsystem design variables are included along with the strictly system level variables. The process is repeated until it has converged to an optimum. As with any linearization technique, move limits must be imposed at each level and these are reduced as the optimization proceeds.

MULTILEVEL OPTIMIZATION PROCESS

EVALUATE SYSTEM LEVEL FUNCTIONS

CREATE LINEAR APPROXIMATION TO SYSTEM LEVEL FUNCTIONS

SOLVE EACH SUBSYSTEM PROBLEM,

INCLUDING LINEARIZED SYSTEM LEVEL CONSTRAINTS

CREATE LINEAR APPROXIMATION TO ALL SUBSYSTEM CONSTRAINTS

SOLVE SYSTEM LEVEL PROBLEM

INCLUDING LINEARIZED SUBSYSTEM CONSTRAINTS

REPEAT TO CONVERGENCE

MOVE LIMITS ARE USED AT EACH LEVEL

FIGURE 5

Figure 6 provides the basic mathematical details of the optimization task at the system level. Here, capital letters indicate strictly system level design variables, objective and constraints, and lower case letters indicate subsystem level variables and constraints. Here, the subsystem variables are included along with the system variables. The subsystem constraints are included in their linearized form. Note that the number of design variables, as well as the number of constraints to be considered here is greatly increased from the number of strictly system level variables and constraints. However, the subsystem constraints are linearized and so are relatively easily dealt with. This is a departure from previous methods which used a cumulative constraint for each subsystem as well as a set of "optimum sensitivities" from the subsystems. The tradeoff is that the functions here are linearized at the expense of an increase in the number of design variables and constraints. However, the need to deal with nonlinear inequality constraints at the subsystem, as well as the need to calculate sensitivities of the optimized subsystems is avoided.

PRESENT METHOD

AT THE SYSTEM LEVEL

DESIGN VARIABLES, $X, x_1, x_2, \dots x_N$

OBJECTIVE, $F(X,x_1,x_2, \dots x_N)$

SYSTEM CONSTRAINTS, $G_J(X,x_1,x_2, \dots x_N)$

SUBSYSTEM CONSTRAINTS, $g_j(X,x_1,x_2, \dots x_N)$

WHERE

$$g_j = g_j^0 + \sum_{i=1}^{nss} \underline{\nabla}_x g_j \cdot (\underline{x}_i - \underline{x}_i^0) + \underline{\nabla}_X g_j \cdot (\underline{X} - \underline{X}^0)$$

FIGURE 6

The basic mathematical details for the subsystem optimization are given in Figure 7. The inputs to the subsystem problem include the boundary conditions, system level variables and system level constraints, all in linearized form. The reason that the system level variables and boundary conditions must be linearized is that these may be functions of the subsystem variables and are not assumed to be constant in the present method. For example, the forces in the members of a structure may be functions of the local variables. Also, if the strictly system variables are functions of the subsystem variables, this must be accounted for. Then, when the approximate system level constraints are calculated, it is first necessary to calculate the approximate values of the system variables and subsystem boundary conditions since the system constraints are functions of these. While this appears to be a bit cumbersome, it must be remembered that these computations are relatively simple matrix operations and so are efficiently performed. Also, if sufficient information is available to calculate these parameters precisely, this may be done to improve the overall efficiency.

<u>PRESENT METHOD</u>

<u>AT EACH SUBSYSTEM</u>

DESIGN VARIABLES x_i

OBJECTIVE, $f(BC, X, x_i)$

SUBSYSTEM CONSTRAINTS $g_j(BC, X, x_i)$

SYSTEM CONSTRAINTS, $G_J(BC, X, x_1, x_2, \ldots x_N)$

WHERE

$$BC = BC^0 + \underline{\nabla}_x BC \cdot (x - x^0)$$

$$X = X^0 + \underline{\nabla}_x X \cdot (x - x^0)$$

$$G_j = G_j^{\,0} + \underline{\nabla}_X G_j \cdot (\underline{X} - \underline{X}^0) + \underline{\nabla}_x G_j \cdot (\underline{x} - \underline{x}^0) + \underline{\nabla}_{BC} G_j \cdot (\underline{BC} - \underline{BC}^0)$$

FIGURE 7

Figure 8 presents the iteration history for the cantilevered beam shown in Figure 4. The problem was also solved by direct application of optimization and those results are shown also. The initial design violated constraints, so the direct method first increased the volume in order to overcome these constraint violations. While it appears from the figure that the multilevel method provided an equivalent convergence rate, it must be remembered that one iteration of the multi-level method consists of optimization of all subsystems followed by a system level optimization. Thus, for this simple example, the direct method is much more efficient computationally. This is generally true for problems that can be solved directly. The value of the multilevel method is for design problems where it is necessary to separate the problem for other reasons.

ITERATION HISTORY FOR CANTILEVER BEAM

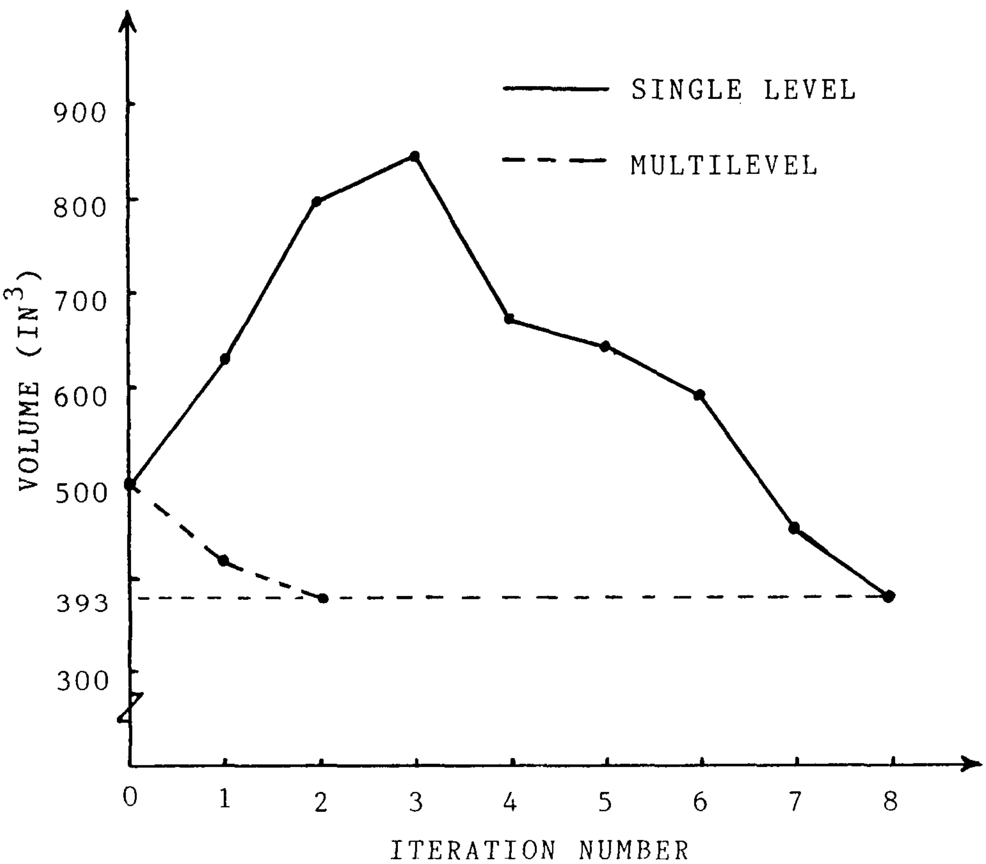


FIGURE 8

The portal frame shown in Figure 9 was designed using the proposed method. This is considered to be a standard test case, and the details of the loads, materials, system and subsystem constraint calculations are presented in Reference 2. The system level constraints are displacement and rotation limits at the joint where the loads are applied. The subsystem constraints included stress, local buckling, and sizing limits. There are three subsystems, being the design of the individual beam elements. The subsystem design variables are the six individual dimensions of the cross-section of each element. The objective function at both the system and subsystem levels is to minimize the volume of material.

Two cases were considered. In the first, thè initial design was well within the feasible region, while in the second, the initial design was quite infeasible. The iteration histories for the two cases are shown in Figures 10 and 11. The multilevel approach did not produce as good an optimum in either case, but did produce a near optimum. There is no clear reason for the differences, although this structure is known to have relative minima.

MULTILEVEL OPTIMIZATION OF A PORTAL FRAME

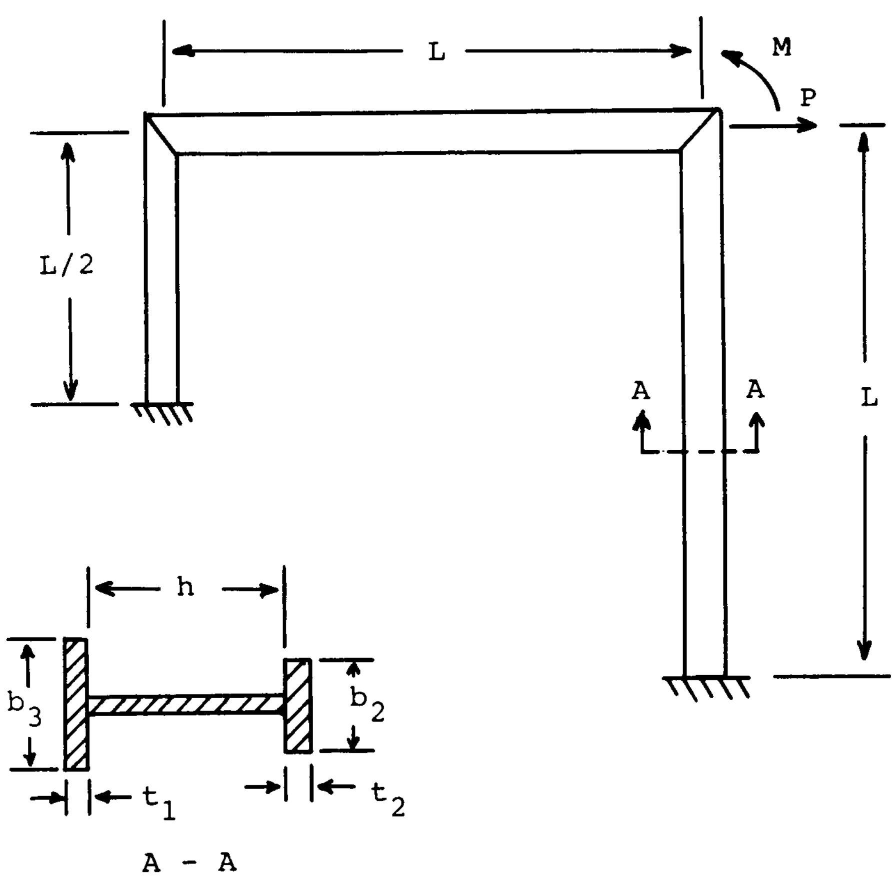


FIGURE 9

ITERATION HISTORIES FOR PORTAL FRAME CASE I

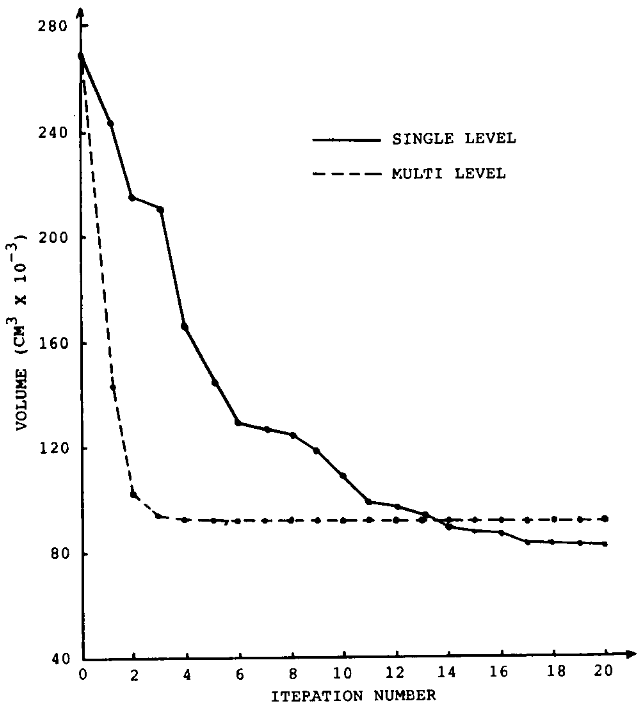


FIGURE 10

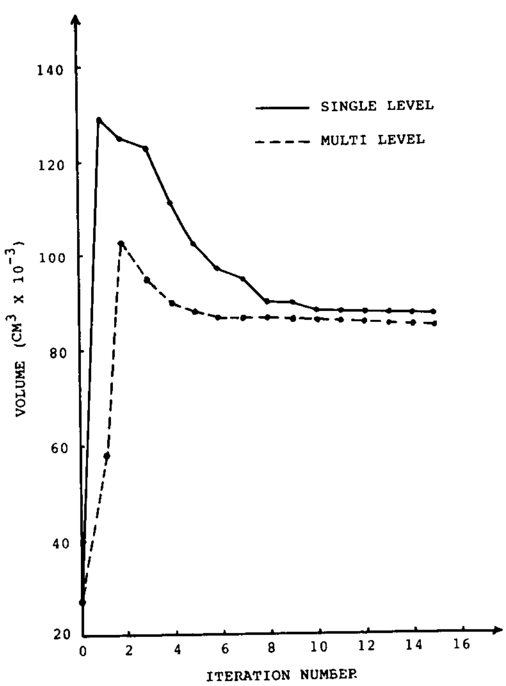


FIGURE 11

The two-bay frame shown in Figure 12 was designed for minimum material using the proposed method. The design variables and material properties for each beam and the subsystem constraints are the same as for the portal frame. The system level constraints are shown in the figure, as well as the loading conditions. Symmetry was used so the system is comprised of four subsystems, being the vertical members of each bay and the floor members of each bay. Each subsystem consists of six design variables for a total of twenty four independent design variables.

The results for single level and multilevel optimization are shown in Figures 13 and 14 for an initially feasible design and an initially infeasible design, respectively.

MULTILEVEL DESIGN OF A TWO-BAY FRAME

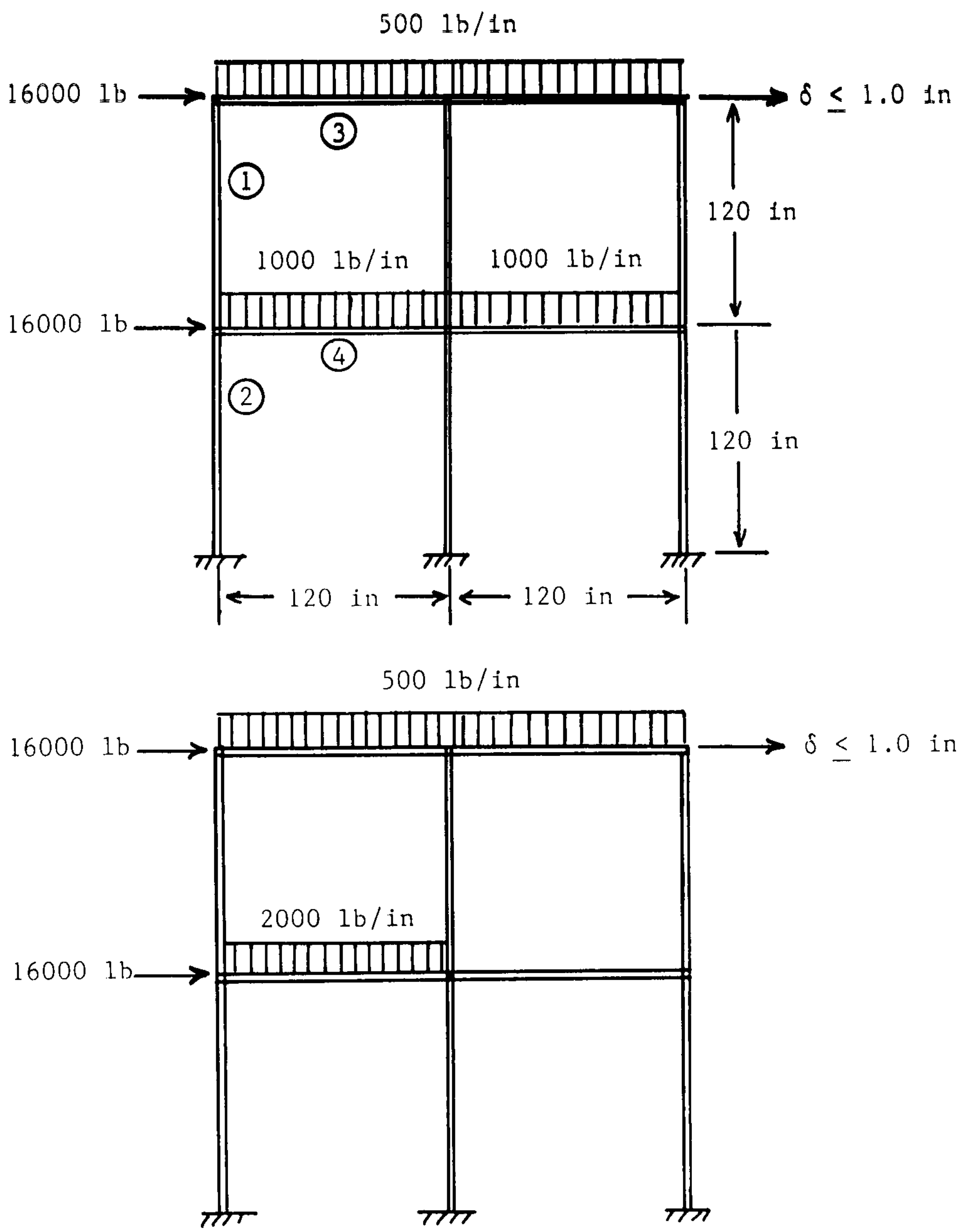


FIGURE 12

ITERATION HISTORIES FOR A TWO-BAY FRAME

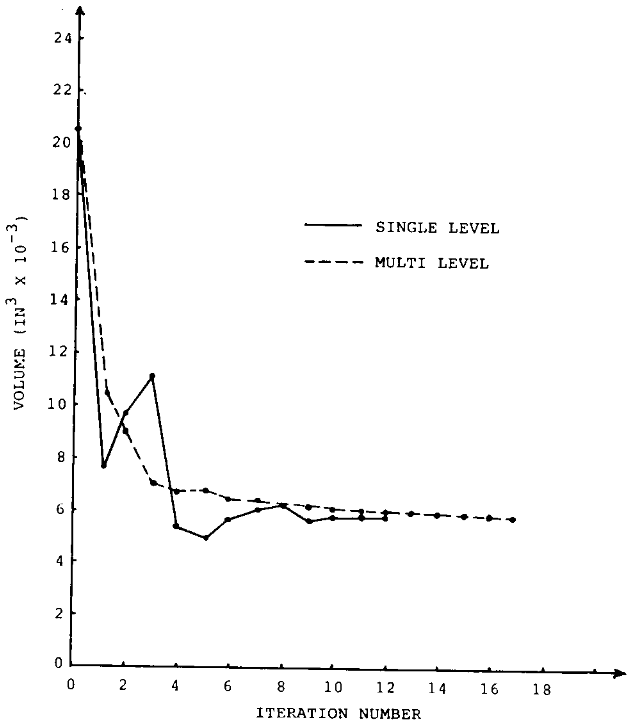


FIGURE 13

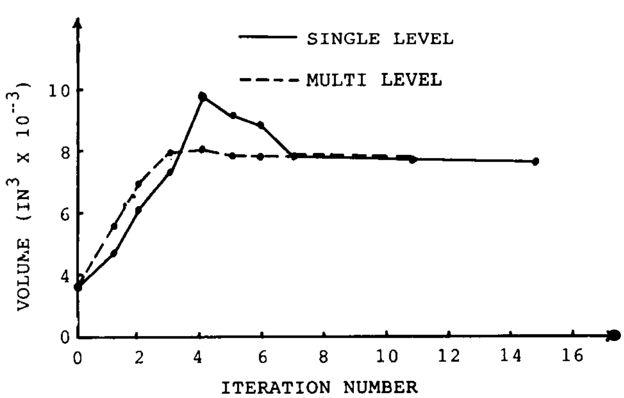


FIGURE 14

Figure 15 lists the advantages and disadvantages of the present approach. A key advantage is that the concept is relatively simple. It does not require the use of nonlinear equality constraints at the subsystem level or the calculation of sensitivities of the subsystem optimum with respect to the system variables as are required in earlier methods. The principal disadvantages are that the number of system level variables is greatly increased and that the quality of the linearizations is important. Regarding this last issue, it should be remembered that the term "linearization" used here does not infer strict linearizations. For example linearizations in reciprocal space may be preferred for structural design problems. The key idea is that the approximations used at each level are explicit.

In summary, it is clear that much research remains to be done before decomposition methods such as this reach the state of reliability that is available in standard structural optimization today. However, for optimization to find widespread use in the multidiscipline environment, it is clear that methods must be developed that will interface with designers with a minimum of disruption to the traditional design environment.

ADVANTAGES OF PRESENT APPROACH

EACH SUBSYSTEM IS SOLVED AS THE ENGINEER CHOOSES

ONLY A SIMPLE SET OF LINEAR CONSTRAINTS MUST BE ADDED

THE SYSTEM AND SUBSYSTEM OBJECTIVES MAY BE DIFFERENT

DEPENDING ON THE NEEDS/MOTIVATIONS OF THE INDIVIDUAL LEVEL

THE SYSTEM LEVEL CONTROLS THE OVERALL OBJECTIVE

THE CONCEPT IS SIMPLE

DISADVANTAGES OF THE PRESENT APPROACH

THE NUMBER OF SYSTEM LEVEL DESIGN VARIABLES IS GREATLY INCREASED

THE QUALITY OF THE LINEARIZATIONS IS IMPORTANT

MOVE LIMITS ARE IMPORTANT

THE METHOD HAS THE SAME OVERALL ADVANTAGES AND

DISADVANTAGES AS SEQUENTIAL LINEAR PROGRAMMING

FIGURE 15

A key issue in the development of "user friendly" multilevel and multidiscipline optimization methods is the user interface. Figure 16 is a general diagram showing the essential components of such a system, and this is the subject of current research at UC Santa Barbara. The control module directs the activities relative to the system and subsystem tasks, as well as basic data management. All data transfer between modules is via a data management system which may be a general system or may be a specialized system for the multidiscipline optimization task. The important aspect of this approach is that the system and subsystems are provided with a specific form of their input and output which is general enough to accommodate the need to operate either independently or within the multidiscipline environment. This only requires a general degree of standardization and the individual disciplines are otherwise free to operate as usual. Also, at the subsystem level, a similar standardization is required to allow the user to perform analysis alone, optimization without an interface to a controlling system, and optimization within the overall system. The purpose of the pilot code being developed is to create such an environment for testing on a variety of multilevel and multidiscipline problems. This is expected to identify more clearly the strengths and weaknesses of the present method as well as identify future research needs.

DATA TRANSFER

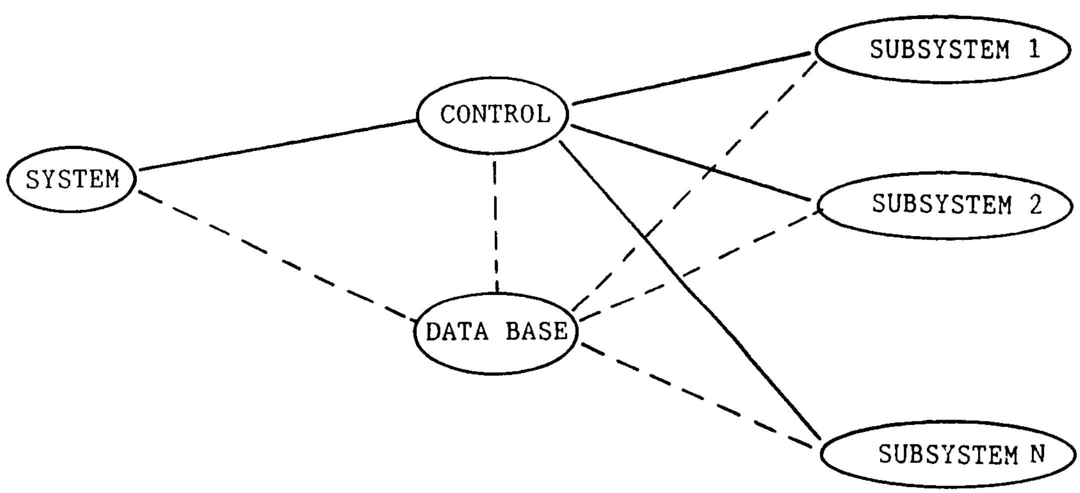


FIGURE 16

REFERENCES

1. Vanderplaats, G. N., Yang, Y. J. and Kim, D. S., "An Efficient Multilevel Optimization Method for Engineering Design, Proc. 1988 AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Williamsburg, Virginia.

2. Sobieszczanski-Sobieski, J., "A linear Decomposition Method for Large Optimization Problems - Blueprint for Development," NASA Technical Memorandum 83,248, February 1982.